Research Paper

Volume : 3 | Issue : 12 | December 2014 • ISSN No 2277 - 8179

WEIGHTED N-FACTOR MARRIAGE PROBLEM USING SMA

MATHEMATICS

KEYWORDS : -Preference Value, Men's Preference value Matrix (PM_M), Women's Preference value Matrix (PM_W), Satisfactory value Matrix ($SM_{M/W}$), Satisfactory level, SMA algorithm, Assignment Technique (Hungarian).

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ABSTRACT

This paper deals with the Marriage problem which was introduced by Gale and Shapley, GS algorithm was used to find solution for it. For a Marriage problem to be solved, only one factor has been used assuming that the preference list is based on a single factor. In this study, the researcher has introduced N-factors with weightage (i.e. Dowry, Height, Weight, Colour and so on) in Marriage problem and Satisfactory Matching Algorithm has been applied to find solution for the marriage problem. The findings were discussed and illustrated with real life examples.

INTRODUCTION

Marriage problem is a problem of matching the elements of two sets (i.e) Men and Women [1]. Matching of men and women for a marriage undergoes many problems and so it has gave significance for the study to find solution. The Stable Marriage problem (SM) introduced by Gale and Shapley consists of two finite equal sized sets. The finite set man denoted by m_i ,choose woman in accordance with their preference. Like-wise the finite set woman denoted by wj ,choose man in accordance with their preference [2]. The GS algorithm [3] propounded by Gale and Shapley states that a stable marriage is an one to one matching of men with women where there is no man-woman pair that prefers each other over their present partner.

Gale and Shapley expressed in their study that only one stable matching exists for every SM. But, in real life there are many different matching for a single Marriage problem.GS algorithm used man or woman optimal as an extreme property for matching to get a best possible partner [4].

In this study, the researcher has introduced Marriage problem with weightage to each factor. An algorithm called Satisfactory Matching Algorithm (SMA) has been used to find solution for the Marriage problem. The solution or findings were illustrated with real life examples for better understanding of the readers.

OBJECTIVES

The main objective of the study is to find out a satisfactory matching of a Marriage problem by considering N-Factors of men and women with weightage to each factor.

NEED FOR THE STUDY

In real life there are many matching problems. Marriage matching has been chosen for the study as it will be apt to explain the situation. Marriage problems discussed earlier have stated that the solutions found were at times, it is men favorable or women favorable solutions. But, in this study, the researcher has narrated a best possible solution for both men and women. This technique can be applied to the situations where optimal solutions to all the members of the set are needed.

LIMITATIONS

The main constraint faced by the researcher for the study is only few factors with weightage were considered for matching the members of the set.

METHODOLOGY

By using assignment technique, Men's and Women's Preference Value Matrix is constructed to find Satisfactory Value Matrix. The Preference Value Matrices are summed up according to their weightage of factor considered for matching and by using SMA algorithm, matching of men and women were found out. Real life problems can also be solved using this technique and is given in this study paper.

N-FACTOR MARRIAGE PROBLEM

The Marriage problem introduced in this paper uses N-Factors and contains two finite equal sets of men and women. The finite set men denoted by m_i ($1 \le i \le n$, n is the number of men) prefer women based on his preferential factors, forming his own preference list. Similarly, the finite set women denoted by w_j ($1 \le j \le n$, n is the number of women) prefer men based on her preferential factors, forming her own preference list. Hence a list of N preference factors can be obtained from men and women. So, it becomes necessary to find out the correct matching between men and women considering their preference factors.

The Related terminologies, Preference value, Men's and Women's Preference value, Men's Preference Value Matrix (PM_M), Women's Preference Value Matrix (PM_W), Satisfactory Value Matrix ($SM_{M/W}$), Satisfactory level, Hungarian method of Assignment model and SMA algorithm are discussed in [5].

Example 1: Consider an instance with three men m_1 , m_2 , m_3 and three women w_1 , w_2 , w_3 and with three factors F_1,F_2,F_3.The members of set of men and women considered factors F_1,F_2,F_3 for 50%,25%,25% respectively as their weightage. The preference lists are given below in the order of preference.

Preference list based on Factor F

m_1: w_2 w_3 w_1	w_1: m_1 m_2 m_3
m_2: w_1 w_2 w_3	w_2: m_1 m_3 m_2
m_3: w2 w_1 w_3	w_3: m_2 m_1 m_3

Preference list based on Factor F_2

m_1: w_1 w_3 w_2	w_1: m_3 m_1 m_2
m_2: w_2 w_1 w_3	w_2: m_1 m_2 m_3
m_3: w_3 w_2 w_1	w_3: m_2 m_3 m_1

Preference list based on Factor F_3

m_1: w_1 w_3 w_2	w_1: m_1 m_2 m_3
m_2: w_1 w_2 w_3	w_2: m_1 m_3 m_2
m_3: w_3 w_1 w_2	w_3: m_2 m_1 m_3

The Satisfactory Value Matrix for Factor F_1 with weightage is

$$SM_{M(F1)} = \begin{array}{c} \\ m_1 \\ m_2 \\ m_3 \end{array} \begin{array}{c} \begin{array}{ccc} w_1 & w_2 & w_3 \end{array} \\ \begin{pmatrix} \frac{2}{3} & \frac{6}{6} & \frac{2}{3} \\ \frac{5}{6} & \frac{1}{2} & \frac{2}{3} \\ \frac{1}{2} & \frac{5}{6} & \frac{1}{3} \end{pmatrix} \end{array}$$

The Satisfactory Value Matrix for Factor F_2 with weightage is

$$SMM(F2) = \begin{array}{c} \\ m_1 \\ m_2 \\ m_3 \end{array} \begin{array}{c} \begin{array}{ccc} w_1 & w_2 & w_3 \end{array} \\ \begin{pmatrix} \frac{5}{12} & \frac{1}{3} & \frac{1}{4} \\ \frac{1}{4} & \frac{5}{12} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{4} & \frac{5}{12} \end{pmatrix} \end{array}$$

The Satisfactory Value Matrix for Factor F_3 with weightage is

$$SM_{M(F3)} = \begin{array}{c} \\ m_1 \\ m_2 \\ m_3 \end{array} \begin{array}{c} \begin{array}{ccc} w_1 & w_2 & w_3 \end{array} \\ \begin{pmatrix} \frac{1}{2} & \frac{1}{3} & \frac{1}{3} \\ \frac{5}{12} & \frac{1}{4} & \frac{1}{3} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{3} \end{pmatrix} \end{array}$$

The resultant Satisfactory Value Matrix is

$$SM_{M(R)} = \begin{array}{c} \\ m_1 \\ m_2 \\ m_3 \end{array} \begin{array}{c} \begin{array}{ccc} w_1 & w_2 & w_3 \end{array} \\ \begin{pmatrix} \frac{19}{12} & \frac{5}{3} & \frac{5}{4} \\ \frac{3}{2} & \frac{7}{6} & \frac{4}{3} \\ \frac{13}{12} & \frac{4}{3} & \frac{13}{12} \end{pmatrix} \end{array}$$

The matching, based on the three Factors with weightage, on applying SMA is (m_1, w_2), (m_2,w_1), (m_3,w_3) and satisfactory value for each pair is given in the following table.

Satisfactory matching	Satisfactory value	Satisfactory level of men	Satisfactory level of women
(m_1,w_2)	5/3	m_1=2/3	w_2=3/3
(m_2,w_1)	3/2	m_2=11/12	w_1=7/12
(m_3,w_3)	13/12	m_3=2/3	w_3=5/12

The matching obtained is the best matching for both the groups and satisfactory level of each group is optimum. The Satisfactory level of men and women are 52.94% and 47.06%.

Example 2: Consider an instance with three men m_1 , m_2 , m_3 and three women w_1 , w_2 , w_3 and with three factors F_1,F_2,F_3.The members of set of men and women considered factors F_1,F_2,F_3 for 40%,40%,20% respectively as their weightage. The preference lists are given below in the order of preference.

Preference list based on Factor F_1

m_1: w_1 w_3 w_2 — w_1: m_3 m_1
m_2: w_2 w_3 — w_2: m_1 m_2m_3
m_3: w_3 w_2 w_1 — w_3: m_2 m_3 m_1

Preference list based on Factor F_2

m_1: w_1 w_3 w_2 — w_1:(m_2 m_1) m_3
m_2: w_1 w_2 w_3 — w_2: m_2 m_3m_1
m_3: w_2 w_3 w_1 — w_3: m_3 m_1 m_2

Preference list based on Factor F_3

m_1: w_2 w_3 w_1 — w_1:m_1(m_2 m_3)
m_2: (w_1 w_3) — w_2: m_1 m_3
m_3: w_2 w_1 — w_3: m_2 m_1

The Satisfactory Value Matrix for Factor F_1 with weightage is

$$SM_{M(F1)} = \begin{array}{c} \\ m_1 \\ m_2 \\ m_3 \end{array} \begin{array}{c} \begin{array}{ccc} w_1 & w_2 & w_3 \end{array} \\ \begin{pmatrix} \frac{2}{3} & \frac{8}{15} & \frac{2}{5} \\ - & \frac{2}{3} & \frac{2}{3} \\ \frac{8}{15} & \frac{2}{5} & \frac{2}{3} \end{pmatrix} \end{array}$$

The Satisfactory Value Matrix for Factor F_2 with weightage is

$$SM_{M(F2)} = \begin{array}{c} \\ m_1 \\ m_2 \\ m_3 \end{array} \begin{array}{c} \begin{array}{ccc} w_1 & w_2 & w_3 \end{array} \\ \begin{pmatrix} \frac{11}{15} & \frac{4}{15} & \frac{8}{15} \\ \frac{11}{15} & \frac{2}{3} & \frac{4}{15} \\ \frac{4}{15} & \frac{2}{3} & \frac{2}{3} \end{pmatrix} \end{array}$$

The Satisfactory Value Matrix for Factor F_3 with weightage is

$$SM_{M(F3)} = \begin{array}{c} \\ m_1 \\ m_2 \\ m_3 \end{array} \begin{array}{c} \begin{array}{ccc} w_1 & w_2 & w_3 \end{array} \\ \begin{pmatrix} \frac{4}{15} & \frac{2}{5} & \frac{4}{15} \\ \frac{4}{15} & - & \frac{11}{30} \\ \frac{7}{30} & \frac{1}{3} & - \end{pmatrix} \end{array}$$

The resultant Satisfactory Value Matrix is

$$SM_{M(R)} = \begin{array}{c} \\ m_1 \\ m_2 \\ m_3 \end{array} \begin{array}{c} \begin{array}{ccc} w_1 & w_2 & w_3 \end{array} \\ \begin{pmatrix} \frac{5}{3} & \frac{6}{5} & \frac{6}{5} \\ 1 & \frac{4}{3} & \frac{13}{10} \\ \frac{31}{30} & \frac{7}{5} & \frac{4}{3} \end{pmatrix} \end{array}$$

Satisfactory matching	Satisfactory value	Satisfactory level of men	Satisfactory level of women
(m_1,w_1)	5/3	m_1=13/15	w_2=4/5
(m_2,w_3)	13/10	m_2=17/30	w_1=11/15
(m_3,w_2)	7/5	m_3=13/15	w_3=8/15

The matching, based on the three Factors with weightage, on applying SMA is (m_1, w_1), (m_2,w_3), (m_3,w_2) and satisfactory value for each pair is given in the following table.

The matching obtained is the best matching for both the groups and satisfactory level of each group is optimum. The Satisfactory level of men and women are 52.67% and 47.33%.

Example 3: Consider an instance with three men m_1 , m_2 , m_3 and three women w_1 , w_2 , w_3 and with three factors F_1,F_2,F_3.The members of set of men and women considered factors F_1,F_2,F_3 for 33.33%,33.33%,33.33% respectively as their weightage. The preference lists are given below in the order of preference.

Preference list based on Factor F_1

m_1: w_2 w_3 w_1 — w_1: m_1 m_2 m_3
m_2: w_1 w_2 w_3 — w_2: m_1 m_3 m_2
m_3: w_2 w_1 w_3 — w_3: m_2 m_1 m_3

Preference list based on Factor F_2

m_1: w_1 w_3 w_2 — w_1: m_3 m_1 m_2
m_2: w_2 w_1 w_3 — w_2: m_1 m_2 m_3
m_3: w_3 w_2 w_1 — w_3: m_2 m_3 m_1

Preference list based on Factor F_3

m_1: w_1 w_3 w_2 — w_1: m_1 m_2 m_3
m_2: w_1 w_2 w_3 — w_2: m_1 m_3 m_2
m_3: w_3 w_1 w_2 — w_3: m_2 m_1 m_3

The Satisfactory Value Matrix for Factor $F_1$with weightage is

$$SM_{M(F1)} = \begin{array}{c} \\ m_1 \\ m_2 \\ m_3 \end{array} \begin{array}{c} \begin{array}{ccc} w_1 & w_2 & w_3 \end{array} \\ \begin{pmatrix} \frac{4}{3} & \frac{6}{3} & \frac{4}{3} \\ \frac{5}{3} & \frac{3}{3} & \frac{4}{3} \\ \frac{3}{3} & \frac{5}{3} & \frac{2}{3} \end{pmatrix} \end{array}$$

The Satisfactory Value Matrix for Factor F_2 with weightage is

$$SM_{M(F2)} = \begin{array}{c} \\ m_1 \\ m_2 \\ m_3 \end{array} \begin{array}{c} \begin{array}{ccc} w_1 & w_2 & w_3 \end{array} \\ \begin{pmatrix} \frac{5}{3} & \frac{4}{3} & \frac{3}{3} \\ \frac{3}{3} & \frac{5}{3} & \frac{4}{3} \\ \frac{4}{3} & \frac{3}{3} & \frac{5}{3} \end{pmatrix} \end{array}$$

The Satisfactory Value Matrix for Factor $F_3$with weightage is

$$SMM(F_3) = \begin{array}{c} \\ m_1 \\ m_2 \\ m_3 \end{array} \begin{array}{c} \begin{array}{ccc} w_1 & w_2 & w_3 \end{array} \\ \begin{pmatrix} \frac{6}{3} & \frac{4}{3} & \frac{4}{3} \\ \frac{5}{3} & \frac{3}{3} & \frac{4}{3} \\ \frac{3}{3} & \frac{3}{3} & \frac{4}{3} \end{pmatrix} \end{array}$$

The resultant Satisfactory Value Matrix is

$$SM_{M(R)} = \begin{array}{c} \\ m_1 \\ m_2 \\ m_3 \end{array} \begin{array}{c} \begin{array}{ccc} w_1 & w_2 & w_3 \end{array} \\ \begin{pmatrix} \frac{15}{3} & \frac{14}{3} & \frac{11}{3} \\ \frac{13}{3} & \frac{11}{3} & \frac{12}{3} \\ \frac{10}{3} & \frac{11}{3} & \frac{11}{3} \end{pmatrix} \end{array}$$

The matching, based on the three Factors with weightage, on applying SMA is (m_1, w_2), (m_2,w_1), (m_3,w_3) and satisfactory value for each pair is given in the following table.

Satisfactory matching	Satisfactory value	Satisfactory level of men	Satisfactory level of women
(m_1,w_2)	14/3	m_1=5/3	w_2=9/3
(m_2,w_1)	13/3	m_2=8/3	w_1=5/3
(m_3,w_3)	11/3	M_3=7/3	w_3=4/3

The above result shows that the satisfactory level of any member in both groups on applying SMA algorithm is 4/3.The matching obtained is the best matching for both the groups and satisfactory level of each group is optimum. The Satisfactory level of men and women are 52.63% and 47.36%.

CONCLUSION

To conclude, in this study, weighted N-Factor Marriage problem has been introduced. SMA algorithm helps to find out the matching between men and women for a marriage considering many factors with weightage. Each factor was studied with a real life example. It was found that both menand women gains high satisfactory level and gets optimal matching. Assignment technique was used to solve the problems with N-Factors. This Technique helps the people to solve matching problems in real life situations and to take absolute decisions in a best possible manner.

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